Ranking Alternatives with Tolerance and Reduction in the Setting of Interval AHP*

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Abstract

Several approaches to the refinement of the dominance relation between alternatives are proposed in the setting of interval AHP in this paper. The approaches are divided into two groups: one uses the tolerance of utility difference and the other uses the reduction of interval priority weights. It is shown that refined dominance relations are obtained relatively easily by solving linear programming problems.

1 Introduction

By the conventional Analytic Hierarchy Process (AHP), alternatives are ranked simply by priority weights estimated from pairwise comparison matrices (PCMs) under multiple criteria [3]. In estimating priority weights, only the pairwise comparison matrices whose consistency degrees are in the acceptable level are treated. Once a priority weight vectors are estimated, the inconsistencies in given pairwise comparison matrices are discarded. From the viewpoint that the decision maker may have vague evaluations, an approach to estimating priority weights by intervals was proposed by Sugihara and Tanaka [5]. Because the intervals estimated by their proposed method do not reflect well the vagueness of the decision maker's evaluations, improved estimation methods have been proposed (see [2]).

Estimating priority weights by intervals is advantageous in making a robust and safe evaluation considering the ambiguity inherent in given PCMs. However, as priority weights are specified only by intervals in these methods, we cannot always rank alternatives clearly. Because the dominance relation between alternatives becomes only a preorder, we cannot judge surely whether an alternative dominates the other for some pairs of alternatives.

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In this paper, we propose several approaches to refining the dominance relation. To rank alternatives clearly under interval priority weights, we introduce two concepts: tolerance in utility difference and reduction of intervals. Tolerance in utility difference assumes that small utility difference is approved. Reduction of intervals assumes that trimming small portion of interval priority weights is accepted. Several conceivable approaches to refining the dominance relation based on those two concepts are proposed. We show that the refined dominance relation can be obtained by solving linear programming problems.

This paper is organized as follows. In the next section, we review the interval AHP and describe a few methods for estimating an interval priority weight vector from a given pairwise comparison matrix. The dominance relation between alternatives is reviewed. In Section 3, approaches to refining the dominance relation are proposed and exemplified. Some concluding remarks are given in Section 4.

2 Interval AHP

We briefly introduce the interval AHP [2, 5] and describe the problem setting of this paper. For the sake of simplicity, we define $N = \{1, 2, \ldots, n\}$ and $N \backslash j = N \backslash \{j\} = \{1, 2, \ldots, j-1, j+1, \ldots, n\}$ for $j \in N$.

As in the conventional AHP [3, 4], we try to estimate the priority weights from a given pairwise comparison matrix A, i.e.,

$$A = \begin{bmatrix} 1 & \cdots & a_{1n} \\ \vdots & a_{ij} & \vdots \\ a_{n1} & \cdots & 1 \end{bmatrix}, \tag{1}$$

where we assume the reciprocity, i.e., $a_{ij} = 1/a_{ji},\ i, j \in N$. Because the $(i, j)^{\mathrm{th}}$ component a_{ij} of A shows the relative importance of the i^{th} item over the j^{th} item. Theoretically, we have $a_{ij} = w_i/w_j,\ i, j \in N$ for priority weights w_i and w_j of i^{th} and j^{th} items. However, because of the vagueness of human judgement, we assume only $a_{ij} \approx w_i/w_j,\ i, j \in N$, where $\approx$ stands for "approximately equals to". Then, in the conventional AHP, $w_i,\ i \in N$ are estimated so as to minimize the errors in A.

In the interval AHP [5], we assume that the decision maker may have a vague priority weight vector whose range can be expressed by an interval priority weight vector $\boldsymbol{W} = (W_1, W_2, \ldots, W_n)^{\mathrm{T}}$ rather than a crisp priority weight vector $\boldsymbol{w}$, where $W_i = [w_i^{\mathrm{L}}, w_i^{\mathrm{R}}],\ i \in N$ and $w_i^{\mathrm{L}} \leq w_i^{\mathrm{R}},\ i \in N$. The inconsistency is assumed to be caused by this vagueness in evaluation of priority weights. Accordingly, we assume that a_{ij} is obtained as w_i/w_j with randomly chosen $w_i \in W_i$ and $w_j \in W_j$. Therefore, $\boldsymbol{W}$ should satisfy $a_{ij} \in [w_i^{\mathrm{L}}/w_j^{\mathrm{R}}, w_i^{\mathrm{R}}/w_j^{\mathrm{L}}],\ i, j \in N,\ i < j$. Let $\mathcal{W}(A)$ be the set of all interval weight vectors $\boldsymbol{W}$ satisfying this condition. Moreover, corresponding to the normality condition of $\boldsymbol{w}$ in the conventional AHP, we require the interval weight vector $\boldsymbol{W}$ to satisfy the normality condition, i.e., $\sum_{j \in N \backslash i} w_j^{\mathrm{R}} +$

$w_i^{\mathrm{L}} \geq 1,\ i \in N$ and $\sum_{j \in N \backslash i} w_j^{\mathrm{L}} + w_i^{\mathrm{R}} \leq 1,\ i \in N$. This condition ensures that for any $w_i^\circ \in W_i$, there exist $w_j \in W_j,\ j \in N \backslash i$ such that $\sum_{j \in N \backslash i} w_j + w_i^\circ = 1$. Let $\mathcal{W}^{\mathrm{N}}$ be the set of all interval weight vectors $\boldsymbol{W}$ satisfying the normality condition.

In the conventional interval AHP [5], interval priority weights $W_i,\ i \in N$ are estimated by solving the following linear programming problem:

$$\underset{\boldsymbol{W}}{\mathrm{minimize}} \{ d(\boldsymbol{W}) \mid \boldsymbol{W} \in \mathcal{W}(A) \cap \mathcal{W}^{\mathrm{N}},\ \epsilon \leq w_i^{\mathrm{L}} \leq w_i^{\mathrm{R}},\ i \in N \}, \tag{2}$$

where ϵ is a sufficiently small positive number and $d : \mathcal{W}^{\mathrm{N}} \to [0, +\infty)$ is defined by

$$d(\boldsymbol{W}) = \sum_{i \in N} (w_i^{\mathrm{R}} - w_i^{\mathrm{L}}). \tag{3}$$

$d(\boldsymbol{W})$ shows the sum of widths of interval priority weights $W_i,\ i \in N$ and it has been considered that the smaller $d(\boldsymbol{W})$ the better estimation. Let $\hat{d}$ be the optimal value to Problem (2).

It is shown that the estimated interval priority weights by (2) do not express well the vagueness of decision maker's evaluation. Therefore, several estimation methods [2] improving the quality of estimated intervals have been proposed. Among them, we consider the maximizing minimum range method which estimates the interval priority weights by the following procedure.

⟨1⟩ Solve the following linear programming problem for each $k \in N$:

$$\underset{\boldsymbol{W}}{\mathrm{minimize}} \{ d_{\bar{k}}(\boldsymbol{W}) \mid \boldsymbol{W} \in \mathcal{W}(A) \cap \mathcal{W}^{\mathrm{N}},\ \epsilon \leq w_i^{\mathrm{L}} \leq w_i^{\mathrm{R}},\ i \in N \}, \tag{4}$$

where $d_k : \mathcal{W}^{\mathrm{N}} \to [0, +\infty)$ is defined by

$$d_k(\boldsymbol{W}) = \sum_{i \in N \backslash k} (w_i^{\mathrm{R}} - w_i^{\mathrm{L}}). \tag{5}$$

Let $\hat{d}_{\bar{k}}$ be the optimal value to Problem (4).

⟨2⟩ Solve the following two linear programming problems for each $k \in N$:

$$\underset{\boldsymbol{W}}{\mathrm{maximize}} \{ w_k^{\mathrm{R}} \mid \boldsymbol{W} \in \mathcal{W}(A) \cap \mathcal{W}^{\mathrm{N}},\ d_{\bar{k}}(\boldsymbol{W}) = \hat{d}_{\bar{k}},\ \epsilon \leq w_i^{\mathrm{L}} \leq w_i^{\mathrm{R}},\ i \in N \}, \tag{6}$$

$$\underset{\boldsymbol{W}}{\mathrm{minimize}} \{ w_k^{\mathrm{L}} \mid \boldsymbol{W} \in \mathcal{W}(A) \cap \mathcal{W}^{\mathrm{N}},\ d_{\bar{k}}(\boldsymbol{W}) = \hat{d}_{\bar{k}},\ \epsilon \leq w_i^{\mathrm{L}} \leq w_i^{\mathrm{R}},\ i \in N \}. \tag{7}$$

Let $\hat{w}_i^{\mathrm{L}}(k)$ and $\hat{w}_i^{\mathrm{R}}(k),\ i \in N$ be values of w_i^{L} and $w_i^{\mathrm{R}},\ i \in N$, respectively, at the obtained optimal solution.

⟨3⟩ The interval weights $\check{W}_j = [\check{w}_j^{\mathrm{L}}, \check{w}_j^{\mathrm{R}}],\ j \in N$ are estimated by the following equations:

$$\check{w}_j^{\mathrm{R}} = \max \left\{ \hat{w}_j^{\mathrm{R}}(k) \mid k \in N \right\}, \quad \check{w}_j^{\mathrm{L}} = \min \left\{ \hat{w}_j^{\mathrm{L}}(k) \mid k \in N \right\}. \tag{8}$$

Because we have $([\hat{w}_1^{\mathrm{L}}(k), \hat{w}_1^{\mathrm{R}}(k)], [\hat{w}_2^{\mathrm{L}}(k), \hat{w}_2^{\mathrm{R}}(k)], \ldots, [\hat{w}_n^{\mathrm{L}}(k), \hat{w}_n^{\mathrm{R}}(k)]) \in \mathcal{W}^{\mathrm{N}}$, $k \in N$, we obtain $([\check{w}_1^{\mathrm{L}}, \check{w}_1^{\mathrm{R}}], [\check{w}_2^{\mathrm{L}}, \check{w}_2^{\mathrm{R}}], \ldots, [\check{w}_n^{\mathrm{L}}, \check{w}_n^{\mathrm{R}}]) \in \mathcal{W}^{\mathrm{N}}$.

Once an interval weight vector $\boldsymbol{W}$ is obtained, we define a dominance relation between alternatives under the assumption that utility values $u_i(o_p)$ of alternatives o_p in view of each criterion are given. We use dominance relation defined by

$$o_p \succsim_O o_q \Leftrightarrow \forall \boldsymbol{w} \in \boldsymbol{W},\ \mathbf{e}^{\mathrm{T}}\boldsymbol{w} = 1;\ \sum_{i \in N}(u_i(o_p) - u_i(o_q))w_i \geq 0, \tag{9}$$

where $\mathbf{e} = (1, 1, \ldots, 1) \in \mathbf{R}^n$. $o_p \succsim_O o_q$ implies that o_p certainly dominates o_q. This dominance relation is only a preorder (reflexive and transitive) because of interval weights. From $\succsim_O$, we obtain a strong dominance relation $\succ_O$ by $o_p \succ o_q \Leftrightarrow o_p \succsim_O o_q$ and $o_q \not\succsim_O o_p$. (9) is rewritten as

$$o_p \succsim_O o_q \Leftrightarrow \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) = \min\left\{ \sum_{i \in N}(u_i(o_p) - u_i(o_q))w_i \,\middle|\, \boldsymbol{w} \in \boldsymbol{W},\ \mathbf{e}^{\mathrm{T}}\boldsymbol{w} = 1 \right\} \geq 0. \tag{10}$$

3 Refining the Dominance Relation

As described above, the dominance relation $\succsim_O$ is usually only a preorder because the dominance relation holds only when an alternative is better than the other for all possible priority weight vectors. The dominance relation indicated by $\succsim_O$ is the result of careful consideration. Therefore, $\succsim_O$ is useful in knowing the robust dominance relation. However, because we may neither rank alternatives nor find the best alternative by using $\succsim_O$, $\succsim_O$ is weak in giving some guidance or instruction for good evaluation and decision. In this section, we investigate the ways to provide some guidance for ranking alternatives. To this end, we propose several methods for ranking alternatives in the presence of interval priority weights. Two approaches are conceivable: one is based on the tolerance of utility differences and the other is based on the reduction of interval priority weights.

3.1 Tolerance approach

3.1.1 By the minimum utility difference

The first approach uses minimum utility differences between alternatives. The objective function of the optimization problem appears in (10) shows the minimum utility difference of alternative o_p from alternative o_q. By exchanging o_p and o_q, we obtain the minimum utility difference of alternative o_q from alternative o_p which shows also minus 1 times of the maximum utility difference of alternative o_p from alternative o_q. Therefore, by solving the optimization problem which is a linear programming problem appears in (10) twice, we obtain the range of utility difference of alternative o_p from alternative o_q as $[\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q), \delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q)]$, where $\delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q) = -\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p)$.

In (10), if the minimum utility difference is non-negative, i.e., $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq 0$, we are sure that o_p is not worse than o_q, i.e., $o_p \succsim_O o_q$. From this definition, we may relax the condition $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq 0$ to $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha$, $\alpha > 0$ is a small number. By this way, we define a relaxed dominance relation as follows:

$$\succsim^{\mathrm{L}}_{\alpha} = \{(o_p, o_q) \mid \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha\}. \tag{11}$$

Namely, we have $o_p \succsim^{\mathrm{L}}_{\alpha} o_q$ if and only if $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha$, where $o_p \succsim^{\mathrm{L}}_{\alpha} o_q$ implies that we are sure that o_p is not very much worse than o_q.

As α increases, $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha$ holds for more ordered pairs (o_p, o_q). When α exceeds a certain value, we may obtain $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha$ and $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p) \geq -\alpha$. Namely, we have $o_p \succsim^{\mathrm{L}}_{\alpha} o_q$ and $o_q \succsim^{\mathrm{L}}_{\alpha} o_p$, i.e., o_p and o_q are indifferent by discarding utility difference α. However, this is not always good if $|\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) - \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p)|$ is sufficiently large comparing to $\max(-\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q), -\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p))$. To avoid this, we modify the definition of $\succsim^{\mathrm{L}}_{\alpha}$ as

$$\succsim^{\mathrm{L}}_{\alpha} = \{(o_p, o_q) \mid \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha \text{ and } \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) > \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p)\}. \tag{12}$$

Moreover, although this modification is applied with a sufficient large α, $\succsim^{\mathrm{L}}_{\alpha}$ cannot always satisfy the transitivity. In other words, the transitive closure $\mathrm{Trcl}(\succsim^{\mathrm{L}}_{\alpha})$ includes indifferences among many alternatives. When all values of $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$ are different, we modify again $\succsim^{\mathrm{L}}_{\alpha}$ by

$$\begin{aligned}\succsim^{\mathrm{L}}_{\alpha} = \{(o_p, o_q) \mid \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) \geq -\alpha \text{ and } (\forall \zeta < \alpha,\ \forall (o_r, o_s) \in \mathrm{Trcl}(\succsim^{\mathrm{L}}_{\zeta} \cup \{(o_p, o_q)\}),\\ \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_r, o_s) \geq -\alpha \text{ or } (o_s, o_r) \notin \mathrm{Trcl}(\succsim^{\mathrm{L}}_{\zeta}))\},\end{aligned} \tag{13}$$

where $\mathrm{Trcl}(\cdot)$ stands for the transitive closure. When k pairs (o_p, o_q) take a same value $\bar{\alpha}$, we introduce some ranking among the k pairs and modify $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$ with $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) + (l-1)\epsilon$, where l shows that pair (o_p, o_q) is ranked as the l-th among the k pairs and ϵ is a very small number. As an example of such an extra ranking, we may order the k pairs in increasing order of $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p)$. As the result, we obtain a weak order $\mathrm{Trcl}(\succsim^{\mathrm{L}}_{\alpha})$ with a sufficient large number α. We select basically the minimum α such that $\mathrm{Trcl}(\succsim^{\mathrm{L}}_{\alpha})$ with $\succsim^{\mathrm{L}}_{\alpha}$ of (13) becomes a weak order.

Example 1. Consider a multiple criteria decision making problem with five criteria $C_1, \ldots, C_5$ and five alternatives $o_1, \ldots, o_5$. We assume the evaluations in view of each criterion is given as in Table 1. Let U be the matrix shown in Table 1. To obtain priority weights of criteria, we asked the decision maker to give a pairwise comparison matrix (PCM). The obtained PCM is shown in Table 2. The consistency index (C.I.) of the PCM is obtained as 0.05209. Because C.I. is smaller than 0.1, we may regard the given PCM is meaningful (see [3]). Applying the maximum eigenvalue method and the geometric mean method used often in the conventional AHP, we obtain the following priority weight vectors, respectively: $\boldsymbol{w}^{\mathrm{E}} = (0.3558, 0.2394, 0.1578, 0.1349, 0.1121)^{\mathrm{T}}$ and $\boldsymbol{w}^{\mathrm{G}} = (0.3468, 0.2424, 0.1599, 0.1392, 0.1117)^{\mathrm{T}}$. The total scores of alternatives are obtained as $U\boldsymbol{w}^{\mathrm{E}} = (0.2192, 0.2087,$

Table 1: Scores of alternatives

	C_1	C_2	C_3	C_4	C_5
o_1	0.25	0.3	0.1	0.15	0.2
o_2	0.2	0.25	0.3	0.1	0.15
o_3	0.15	0.2	0.25	0.3	0.1
o_4	0.1	0.15	0.2	0.25	0.3
o_5	0.3	0.1	0.15	0.2	0.25

Table 2: Pairwise comparison matrix

	C_1	C_2	C_3	C_4	C_5
C_1	1	1	2	2	6
C_2	1	1	1	2	2
C_3	1/2	1	1	1	1
C_4	1/2	1/2	1	1	1
C_5	1/6	1/2	1	1	1

Table 3: $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$

	o_1	o_2	o_3	o_4	o_5
o_1	–	−0.018182	−0.031031	0.008553	−0.003947
o_2	−0.01875	–	−0.033333	−0.010197	−0.022697
o_3	−0.0375	−0.01875	–	0.004546	−0.041447
o_4	−0.077083	−0.058333	−0.039583	–	−0.057143
o_5	−0.028947	−0.045833	−0.027083	0.0125	–

$0.1924, 0.1704, 0.2094)^{\mathrm{T}}$ and $U\boldsymbol{w}^{\mathrm{G}} = (0.2186, 0.2086, 0.1934, 0.1713, 0.2080)^{\mathrm{T}}$. Then, we obtain $o_1 \succ^{\mathrm{E}} o_5 \succ^{\mathrm{E}} o_2 \succ^{\mathrm{E}} o_3 \succ^{\mathrm{E}} o_4$ and $o_1 \succ^{\mathrm{G}} o_2 \succ^{\mathrm{G}} o_3 \succ^{\mathrm{G}} o_5 \succ^{\mathrm{E}} o_4$, respectively. We note that the orders are different between the maximum eigenvalue method and the geometric mean method although C.I. is small enough.

Now we apply the interval AHP. Estimating the interval priority weights by the maximizing minimum range method, we obtain $\boldsymbol{W} = ([0.25, 0.4286], [0.1842, 0.3158], [0.125, 0.2727], [0.125, 0.3333], [0.04167, 0.1818])^{\mathrm{T}}$. $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$ are obtained as shown in Table 3. Then we obtain only $o_1 \succ_O o_4$, $o_3 \succ_O o_4$ and $o_5 \succ_O o_4$ when $\alpha = 0$. Setting $\alpha = 0.031031$ or larger, we obtain a weak order defined by $\mathrm{Trcl}(\succsim^{\mathrm{L}}_{\alpha})$. When $\alpha = 0.031031$, we have

$$\succsim^{\mathrm{L}}_{\alpha} = \{(o_1, o_4), (o_3, o_4), (o_5, o_4), (o_1, o_5), (o_2, o_4), (o_1, o_2), (o_3, o_2), (o_2, o_5), (o_1, o_3)\}. \tag{14}$$

By the transitive closure of this relation, we obtain $o_1 \hat{\succsim}^{\mathrm{L}}_{\alpha} o_3 \hat{\succsim}^{\mathrm{L}}_{\alpha} o_2 \hat{\succsim}^{\mathrm{L}}_{\alpha} o_5 \hat{\succsim}^{\mathrm{L}}_{\alpha} o_4$, where $\hat{\succsim}^{\mathrm{L}}_{\alpha} = \mathrm{Trcl}(\succsim^{\mathrm{L}}_{\alpha})$. □

We note that in the obtained refinement of dominance relation we understand $o_p \succsim^{\mathrm{L}}_{\alpha} o_q$ is accepted more easily if $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$ is larger.

3.1.2 By the center value of the utility difference

In the previous subsection, we refined the dominance relation by the minimum value of the utility difference. However, the minimum value can be considerably small if the width of the interval of utility difference is large even if the location of the interval is around zero. For example, in Example 1, the location of the utility difference between o_1 and o_2 is around zero because the difference between

$\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_1, o_2)$ and $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_2, o_1)$ is very small. The location can be seen by the center value of the interval. Then, in this subsection, we consider a refinement by using the center values of utility difference intervals. Because the range of the utility difference of alternative o_p from alternative o_q is obtained as $[\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q), \delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q)]$. Then the center value of the utility difference of alternative o_p from alternative o_q can be obtained by

$$\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q) = \frac{1}{2}(\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) + \delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q)). \tag{15}$$

As we have $\delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q) = -\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_q, o_p)$, we obtain $\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q) = -\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_q, o_p)$. Because of this special relation, each ordered pair (o_p, o_q) such that $\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q) \geq 0$ is a candidate of the refined dominance relation $o_p \succsim^{\mathrm{C}} o_q$. We note that $o_p \succsim_O o_q$ is always a candidate of $o_p \succsim^{\mathrm{C}} o_q$. However, unfortunately, the dominance relation composed of the candidates does not always become a weak order. To overcome this inadequacy, we apply the same idea as $\succsim^{\mathrm{L}}_{\alpha}$. Namely, for $\eta \geq 0$, we define

$$\begin{aligned}\succsim^{\mathrm{C}}_{\eta} = \{(o_p, o_q) \mid \delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q) \geq -\eta \text{ and } (\forall \zeta < \eta,\ \forall (o_r, o_s) \in \mathrm{Trcl}(\succsim^{\mathrm{C}}_{\zeta} \cup \{(o_p, o_q)\}),\\ \delta^{\mathrm{C}}_{\boldsymbol{W}}(o_r, o_s) \geq -\eta \text{ or } (o_s, o_r) \notin \mathrm{Trcl}(\succsim^{\mathrm{C}}_{\zeta}))\}.\end{aligned} \tag{16}$$

We select η by the minimum value such that $\mathrm{Trcl}(\succsim^{\mathrm{C}}_{\eta})$ becomes a weak order.

Applying the approach of $\succsim^{\mathrm{C}}_{\eta}$ in Example 1, we obtain the same refined weak order $o_1 \hat{\succsim}^{\mathrm{C}}_{\eta} o_3 \hat{\succsim}^{\mathrm{C}}_{\eta} o_2 \hat{\succsim}^{\mathrm{C}}_{\eta} o_5 \hat{\succsim}^{\mathrm{C}}_{\eta} o_4$ with $\eta = 0.0032345$, where $\hat{\succsim}^{\mathrm{C}}_{\eta} = \mathrm{Trcl}(\succsim^{\mathrm{C}}_{\eta})$. However, $\succsim^{\mathrm{C}}_{\eta}$ is different from $\succsim^{\mathrm{L}}_{\alpha}$ in (14), i.e., we have

$$\succsim^{\mathrm{C}}_{\eta} = \{(o_1, o_4), (o_5, o_4), (o_2, o_4), (o_3, o_4), (o_1, o_5), (o_2, o_5), (o_3, o_2), (o_1, o_3)\}. \tag{17}$$

As is shown in (17), (o_1, o_2) does not appear in $\succsim^{\mathrm{C}}_{\eta}$ of (17) while it appears in $\succsim^{\mathrm{L}}_{\alpha}$ of (14). This exemplifies the case where $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_1, o_2)$ is rather large but $\delta^{\mathrm{R}}_{\boldsymbol{W}}(o_1, o_2) - \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_1, o_2)$ is small.

When we use the center values, a simpler approach is conceivable. It utilizes the average degree of dominance

$$avdd(o_p) = \frac{1}{n-1} \sum_{q \in N \setminus p} \delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q). \tag{18}$$

The larger the average degree of dominance is, the larger we consider its utility is. Therefore, we may rank alternatives by $avdd(o_p)$. This order is denoted by $\bar{\succsim}^{\mathrm{C}}$. We note that $\sum_{p \in N} avdd(o_p) = 0$. Therefore, we may regard o_p as a preferable alternative if $avdd(o_p) > 0$.

Applying this approach to Example 1, we obtain $avdd(o_1) = 0.058837$, $avdd(o_2) = 0.028061$, $avdd(o_3) = 0.018940$, $avdd(o_4) = -0.123772$ and $avdd(o_5) = 0.017936$. Then the refined dominance relations is obtained as $o_1 \bar{\succsim}^{\mathrm{C}} o_2 \bar{\succsim}^{\mathrm{C}} o_3 \bar{\succsim}^{\mathrm{C}} o_5 \bar{\succsim}^{\mathrm{C}} o_4$. We note that $avdd(o_1)$ and $avdd(o_2)$ are sufficiently different although $\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_1, o_2)$ is very small.

Table 4: $prt(o_p, o_q)$

	o_1	o_2	o_3	o_4	o_5
o_1	–	0.507690	0.547198	1	0.880009
o_2	0.492310	–	0.360002	0.851204	0.668802
o_3	0.452802	0.639998	–	1	0.395199
o_4	0	0.148796	0	–	0
o_5	0.119991	0.331198	0.604801	1	–

3.2 By the positive ratio of the interval utility difference

In the approach using the center value of interval utility difference, the width of interval utility difference is not taken care at all. Third approach is to take care of the location and the width of interval utility difference. We consider the positive ratio of the interval utility difference. Namely, we calculate the ratio of positive region to the whole range of possible utility differences, i.e.,

$$prt(o_p, o_q) = \frac{\max(\delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q), 0) - \max(\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q), 0)}{\delta^{\mathrm{R}}_{\boldsymbol{W}}(o_p, o_q) - \delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)} \tag{19}$$

We note that we have $prt(o_p, o_q) = 1$ if and only if $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q) > 0$, and we have $prt(o_p, o_q) > 0.5$ if and only if $\delta^{\mathrm{C}}_{\boldsymbol{W}}(o_p, o_q) > 0$.

We apply the same idea as $\succsim^{\mathrm{L}}_{\alpha}$ and $\succsim^{\mathrm{C}}_{\eta}$. Then, for $\rho \geq 0.5$, we define

$$\begin{aligned} \succsim^{\mathrm{P}}_{\rho} = \{(o_p, o_q) \mid prt(o_p, o_q) \geq \rho \text{ and } (\forall \zeta < \rho,\ \forall (o_r, o_s) \in \mathrm{Trcl}(\succsim^{\mathrm{P}}_{\zeta} \cup \{(o_p, o_q)\}), \\ prt(o_r, o_s) \geq \rho \text{ or } (o_s, o_r) \notin \mathrm{Trcl}(\succsim^{\mathrm{P}}_{\zeta}))\}. \end{aligned} \tag{20}$$

As we decrease ρ, we obtain a weak order $\mathrm{Trcl}(\succsim^{\mathrm{P}}_{\rho})$. We select ρ basically with the minimum value such that $\mathrm{Trcl}(\succsim^{\mathrm{P}}_{\rho})$ becomes a weak order.

Example 2. Consider the same pairwise comparison matrix and normalized interval weight vector $\boldsymbol{W}$ as in Example 1. Based on $\delta^{\mathrm{L}}_{\boldsymbol{W}}(o_p, o_q)$ values shown in Table 3, we obtain $prt(o_p, o_q)$ as shown in Table 4. Applying the approach of $\succsim^{\mathrm{P}}_{\rho}$, from Table 4, we obtain a refined weak order, $o_1 \hat{\succsim}^{\mathrm{P}}_{\rho} o_3 \hat{\succsim}^{\mathrm{P}}_{\rho} o_2 \hat{\succsim}^{\mathrm{P}}_{\rho} o_5 \hat{\succsim}^{\mathrm{P}}_{\rho} o_4$ with $\rho = 0.547198$ or larger. When $\rho = 0.547198$,

$$\succsim^{\mathrm{P}}_{\rho} = \{(o_1, o_4), (o_3, o_4), (o_5, o_4), (o_1, o_5), (o_2, o_4), (o_2, o_5), (o_3, o_2), (o_1, o_3)\}. \tag{21}$$

Pair (o_1, o_2) appear neither in $\succsim^{\mathrm{P}}_{\rho}$.

3.3 Reduction approach

Several approaches to refining dominance relation $\succsim_O$ based on utility difference have proposed in the previous subsection. As another approach, a method based on interval weight reduction is conceivable (see [1]). In this subsection, the approach based on the reduction of interval weights is described.

Let $\boldsymbol{V} = (V_1, V_2, \dots, V_n)^{\mathrm{T}}$ be a reduced interval priority weight vector of a given interval weight vector $\boldsymbol{W}$ such that $w_i^{\mathrm{L}} \le v_i^{\mathrm{L}} \le v_i^{\mathrm{R}} \le w_i^{\mathrm{R}}$, $i \in N$, where $V_i = [v_i^{\mathrm{L}}, v_i^{\mathrm{R}}]$, $i \in N$. For o_p to dominate o_q, the reduced interval weight vector $\boldsymbol{V} \subseteq \boldsymbol{W}$ should satisfy

$$\min \left\{ \sum_{i \in N} (u_i(o_p) - u_i(o_q)) v_i \;\middle|\; v_i^{\mathrm{L}} \le v_i \le v_i^{\mathrm{R}},\ i \in N,\ \sum_{i \in N} v_i = 1 \right\} \ge 0. \tag{22}$$

Let us define the following three index sets of N:

$$I^+(o_p, o_q) = \{ i \in N \mid u_i(o_p) - u_i(o_q) > 0 \}, \tag{23}$$
$$I^-(o_p, o_q) = \{ i \in N \mid u_i(o_p) - u_i(o_q) < 0 \}, \tag{24}$$
$$I^0(o_p, o_q) = \{ i \in N \mid u_i(o_p) - u_i(o_q) = 0 \}. \tag{25}$$

v_i^{L} for $i \in I^+(o_p, o_q)$ and v_i^{R} for $i \in I^-(o_p, o_q)$ tend to minimize the objective function of the minimization problem in (22). Indeed, if we drop the constraint $\sum_{i \in N} v_i = 1$ from the minimization problem in (22), v_i^{L} for $i \in I^+(o_p, o_q)$ and v_i^{R} for $i \in I^-(o_p, o_q)$ attain the minimum. From this fact, we take care of the changes of the lower bounds of interval priority weights when $u_i(o_p) > u_i(o_q)$ and the changes of the upper bounds of interval priority weights when $u_i(o_p) < u_i(o_q)$.

Then we define the ambiguity reduction rates of $\boldsymbol{V} \subseteq \boldsymbol{W}$ in the following two ways:

$$\begin{aligned}
&\text{individual ambiguity reduction rate:}\\
&\quad ir(o_p, o_q) = \min \left(\min_{i \in I^-(o_p, o_q)} \frac{w_i^{\mathrm{R}} - v_i^{\mathrm{R}}}{w_i^{\mathrm{R}} - w_i^{\mathrm{L}}}, \min_{i \in I^+(o_p, o_q)} \frac{v_i^{\mathrm{L}} - w_i^{\mathrm{L}}}{w_i^{\mathrm{R}} - w_i^{\mathrm{L}}} \right),\\
&\text{total ambiguity reduction rate:}\\
&\quad tr(o_p, o_q) = \frac{\displaystyle\sum_{i \in I^-(o_p, o_q)} (w_i^{\mathrm{R}} - v_i^{\mathrm{R}}) + \sum_{i \in I^+(o_p, o_q)} (v_i^{\mathrm{L}} - w_i^{\mathrm{L}})}{\displaystyle\sum_{i \in I^-(o_p, o_q) \cup I^+(o_p, o_q)} (w_i^{\mathrm{R}} - w_i^{\mathrm{L}})}.
\end{aligned} \tag{26}$$

In this paper, we will find, for each ordered pair (o_p, o_q) of alternatives, the reduced interval priority weight vector $\boldsymbol{V}(o_p, o_q)$ which maximizes an ambiguity reduction rate $rd(o_p, o_q)$ such that $\forall \boldsymbol{V}' \supseteq \boldsymbol{V}(o_p, o_q)$ satisfying $\boldsymbol{V}' \subseteq \boldsymbol{W}$, $\boldsymbol{V}' \in \mathcal{W}^{\mathrm{N}}$ and $\delta_{\boldsymbol{V}'}^{\mathrm{L}} \ge 0$. In other words, we maximize an ambiguity reduction rate $rd(o_p, o_q)$ such that $\exists \boldsymbol{v} = (v_1, \dots, v_n)^{\mathrm{T}} \in \boldsymbol{V}(o_p, o_q)$ satisfying $\mathbf{e}^{\mathrm{T}} \boldsymbol{v} = 1$ and $\sum_{i \in N} (u_i(o_p) - u_i(o_q)) v_i \le 0$. For $rd(o_p, o_q)$, we consider $ir(o_p, o_q)$ and $tr(o_p, o_q)$.

The maximum $ir(o_p, o_q)$ and $tr(o_p, o_q)$ as well as their corresponding $\boldsymbol{V}(o_p, o_q)$

can be obtained by solving the following linear programming problems, respectively:

$$\begin{aligned}
\text{maximize}\quad & r, \\
\text{sub. to}\quad & \sum_{i\in N}(u_i(o_p)-u_i(o_q))v_i \le 0, \\
& \sum_{i\in N} v_i = 1,\ v_i^{\mathrm{L}} \le v_i \le v_i^{\mathrm{R}},\ i\in N, \\
& v_i^{\mathrm{L}} - (w_i^{\mathrm{R}}-w_i^{\mathrm{L}})r \ge w_i^{\mathrm{L}},\ v_i^{\mathrm{R}} \le w_i^{\mathrm{R}},\ i\in I^+(o_p,o_q), \\
& v_i^{\mathrm{R}} + (w_i^{\mathrm{R}}-w_i^{\mathrm{L}})r \le w_i^{\mathrm{R}},\ v_i^{\mathrm{L}} \ge w_i^{\mathrm{L}},\ i\in I^-(o_p,o_q), \\
& v_i^{\mathrm{L}} \ge w_i^{\mathrm{L}},\ v_i^{\mathrm{R}} \le w_i^{\mathrm{R}},\ i\in I^0(o_p,o_q), \\
& v_i^{\mathrm{L}} + \sum_{j\in N\setminus j} v_i^{\mathrm{R}} \ge 1,\ v_i^{\mathrm{R}} + \sum_{j\in N\setminus j} v_i^{\mathrm{L}} \le 1,\ i\in N, \\
& v_i^{\mathrm{L}} \ge \epsilon,\ i\in N,\ r\ge 0,
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
\text{maximize}\quad & \sum_{i\in I^+(o_p,o_q)\cup I^-(o_p,o_q)} r_i \Bigg/ \sum_{i\in I^+(o_p,o_q)\cup I^-(o_p,o_q)} (w_i^{\mathrm{R}}-w_i^{\mathrm{L}}), \\
\text{sub. to}\quad & \sum_{i\in N}(u_i(o_p)-u_i(o_q))v_i \le 0, \\
& \sum_{i\in N} v_i = 1,\ v_i^{\mathrm{L}} \le v_i \le v_i^{\mathrm{R}},\ i\in N, \\
& v_i^{\mathrm{L}} - r_i \ge w_i^{\mathrm{L}},\ v_i^{\mathrm{R}} \le w_i^{\mathrm{R}},\ i\in I^+(o_p,o_q), \\
& v_i^{\mathrm{R}} + r_i \le w_i^{\mathrm{R}},\ v_i^{\mathrm{L}} \ge w_i^{\mathrm{L}},\ i\in I^-(o_p,o_q), \\
& v_i^{\mathrm{L}} \ge w_i^{\mathrm{L}},\ v_i^{\mathrm{R}} \le w_i^{\mathrm{R}},\ i\in I^0(o_p,o_q), \\
& v_i^{\mathrm{L}} + \sum_{j\in N\setminus j} v_i^{\mathrm{R}} \ge 1,\ v_i^{\mathrm{R}} + \sum_{j\in N\setminus j} v_i^{\mathrm{L}} \le 1,\ i\in N, \\
& v_i^{\mathrm{L}} \ge \epsilon,\ i\in N,\ r_i\ge 0,\ i\in I^+(o_p,o_q)\cup I^-(o_p,o_q).
\end{aligned} \tag{28}$$

We obtain $ir(o_p,o_q)$ and $tr(o_p,o_q)$ by optimal values of Problems (27) and (28), respectively. For each of those problems, the reduced interval priority weight vector $\boldsymbol{V}(o_p,o_q)=(V_1(o_p,o_q),\dots,V_n(o_p,o_q))^{\mathrm{T}}$ are obtained by $V_i(o_p,o_q)=[v_i^{\mathrm{L}},v_i^{\mathrm{R}}]$, $i\in N$ from an optimal solution.

For $ir(o_p,o_q)$ and $ir(o_q,o_p)$, we have $ir(o_p,o_q)+ir(o_q,o_p)\le 1$, and for $tr(o_p,o_q)$ and $tr(o_q,o_p)$, we have $tr(o_p,o_q)+tr(o_q,o_p)\le 1$. These can be proven as follows: we show $tr(o_p,o_q)+tr(o_q,o_p)\le 1$. Let $\boldsymbol{r}^*=(r_1^*,\dots,r_n^*)^{\mathrm{T}}$, $\boldsymbol{v}^{\mathrm{L}*}=(v_1^{\mathrm{L}*},\dots,v_n^{\mathrm{L}*})^{\mathrm{T}}$ and $\boldsymbol{v}^{\mathrm{R}*}=(v_1^{\mathrm{R}*},\dots,v_n^{\mathrm{R}*})^{\mathrm{T}}$ compose an optimal solution to Problem (28). We have $tr(o_p,o_q)=\sum_{i\in I^+(o_p,o_q)\cup I^-(o_p,o_q)} r_i$, $v_i^{\mathrm{L}*}=w_i^{\mathrm{L}*}+r_i$, $i\in I^+(o_p,o_q)$, $v_i^{\mathrm{R}*}=w_i^{\mathrm{R}}+r_i$, $i\in I^-(o_p,o_q)$ and $\forall \boldsymbol{v}=(v_1,\dots,v_n)^{\mathrm{T}}$ such that $\boldsymbol{v}^{\mathrm{L}}\le \boldsymbol{v}\le \boldsymbol{v}^{\mathrm{R}}$, we have $\sum_{i\in N}(u_i(o_p)-u_i(o_q))v_i\le 0$. From the last property, we obtain

$$tr(o_q,o_p) \le \frac{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)} (w_i^{\mathrm{R}}-v_i^{\mathrm{R}*})}{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)} (w_i^{\mathrm{R}}-w_i^{\mathrm{L}})}. \tag{29}$$

Because $I^+(o_p,o_q)=I^-(o_q,o_p)$ holds. Then we obtain $w_i^{\mathrm{R}}-v_i^{\mathrm{R}*}\le(w_i^{\mathrm{R}}-w_i^{\mathrm{L}})-r_i$, $i\in I^+(o_p,o_q)$ and $v_i^{\mathrm{L}*}-w_i^{\mathrm{L}}\le(w_i^{\mathrm{R}}-w_i^{\mathrm{L}})-r_i$, $i\in I^-(o_p,o_q)$. Therefore, we obtain

$$\begin{aligned}\frac{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)}(w_i^{\mathrm{R}}-v_i^{\mathrm{R}*})}{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)}(w_i^{\mathrm{R}}-w_i^{\mathrm{L}})} &\le \frac{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)}(w_i^{\mathrm{R}}-w_i^{\mathrm{L}})-r_i}{\displaystyle\sum_{i\in I^+(o_q,o_p)\cup I^-(o_q,o_p)}(w_i^{\mathrm{R}}-w_i^{\mathrm{L}})}\\ &=1-tr(o_p,o_q). \end{aligned}\tag{30}$$

From (29) and (30), we conclude $tr(o_q,o_p)+tr(o_p,o_q)\le 1$. The other can be proven in the same way.

The smaller $ir(o_p,o_q)$ and $tr(o_p,o_q)$ are, the more acceptable $o_p\succsim o_q$ is. Then we refine $\succsim_O$ by accepting $o_p\succsim o_q$ with small $ir(o_p,o_q)$ and/or $tr(o_p,o_q)$ values. Then we apply the same idea as $\succsim_\alpha^{\mathrm{L}}$ to obtain a refined dominance relation using $ir(o_p,o_q)$ or $tr(o_p,o_q)$. Namely, we obtain

$$\begin{aligned}\succsim_\tau^{\mathrm{ir}}=\{(o_p,o_q)\mid ir(o_p,o_q)\le\tau \text{ and } (\forall\zeta<\tau,\ \forall(o_r,o_s)\in\mathrm{Trcl}(\succsim_\zeta^{\mathrm{ir}}\cup\{(o_p,o_q)\}),\\ ir(o_r,o_s)\le\tau \text{ or } (o_s,o_r)\notin\mathrm{Trcl}(\succsim_\zeta^{\mathrm{ir}}))\},\end{aligned}\tag{31}$$

$$\begin{aligned}\succsim_\upsilon^{\mathrm{tr}}=\{(o_p,o_q)\mid tr(o_p,o_q)\le\upsilon \text{ and } (\forall\zeta<\upsilon,\ \forall(o_r,o_s)\in\mathrm{Trcl}(\succsim_\zeta^{\mathrm{tr}}\cup\{(o_p,o_q)\}),\\ tr(o_r,o_s)\le\upsilon \text{ or } (o_s,o_r)\notin\mathrm{Trcl}(\succsim_\zeta^{\mathrm{tr}}))\}.\end{aligned}\tag{32}$$

Taking their transitive closures, we obtain weak orders among alternatives. τ and υ are defined by the minimum values such that their trensitive clusures become weak orders.

Example 3. Consider the same pairwise comparison matrix and normalized interval weight vector $\boldsymbol{W}$ as in Example 1. We obtain $ir(o_p,o_q)$ and $tr(o_p,o_q)$ as shown in Tables 5 and 6. Then, with $\tau=0.339898$ and $\upsilon=0.463964$, we obtain

$$\begin{aligned}\succsim_\tau^{\mathrm{ir}}=\{&(o_1,o_4),(o_3,o_4),(o_5,o_4),(o_2,o_4),(o_1,o_5),\\ &(o_1,o_3),(o_1,o_2),(o_2,o_5),(o_2,o_3),(o_5,o_3)\},\end{aligned}\tag{33}$$

$$\begin{aligned}\succsim_\upsilon^{\mathrm{tr}}=\{&(o_1,o_4),(o_3,o_4),(o_5,o_4),(o_2,o_4),(o_1,o_2),\\ &(o_1,o_3),(o_2,o_3),(o_5,o_2),(o_5,o_1)\}.\end{aligned}\tag{34}$$

Eventually, we obtain refined weak orders for $\tau\ge 0.339898$ and $\upsilon\ge 0.463964$, $o_1\mathrel{\hat{\succsim}_\tau^{\mathrm{ir}}}o_2\mathrel{\hat{\succsim}_\tau^{\mathrm{ir}}}o_5\mathrel{\hat{\succsim}_\tau^{\mathrm{ir}}}o_3\mathrel{\hat{\succsim}_\tau^{\mathrm{ir}}}o_4$ and $o_5\mathrel{\hat{\succsim}_\upsilon^{\mathrm{tr}}}o_1\mathrel{\hat{\succsim}_\upsilon^{\mathrm{tr}}}o_2\mathrel{\hat{\succsim}_\upsilon^{\mathrm{tr}}}o_3\mathrel{\hat{\succsim}_\upsilon^{\mathrm{tr}}}o_4$, where $\hat{\succsim}_\tau^{\mathrm{ir}}$ and $\hat{\succsim}_\upsilon^{\mathrm{tr}}$ are transitive closures of $\succsim_\tau^{\mathrm{ir}}$ and $\succsim_\upsilon^{\mathrm{tr}}$, respectively.

4 Concluding Remarks

As shown in Example 1, the dominance relation obtained by the conventional AHP is not always unswerving even when the given pairwise comparison matrix is

Table 5: $ir(o_p, o_q)$

	o_1	o_2	o_3	o_4	o_5
o_1	–	0.302326	0.275643	0	0.119997
o_2	0.507693	–	0.332965	0.0890831	0.325053
o_3	0.421277	0.360001	–	0	0.339984
o_4	1	0.5	1	–	1
o_5	0.382839	0.347844	0.339898	0	–

Table 6: $tr(o_p, o_q)$

	o_1	o_2	o_3	o_4	o_5
o_1	–	0.337131	0.409472	0	0.536036
o_2	0.662869	–	0.412291	0.193388	0.538855
o_3	0.590528	0.587709	–	0	0.466514
o_4	1	0.806612	1	–	1
o_5	0.463964	0.461145	0.533486	0	–

sufficiently consistent. We showed that various weak orders are obtained depending on the idea of refinement of dominance relation. In ranking alternatives, those possible weak orders should be considered and the dominance relations obtained by the proposed approach should be interpreted in the real world setting. Moreover, we may combine the proposed tolerance and reduction approaches. These would be included in future topics.

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